Electric and Electronic Engineering

Introduction

Computer engineering is concerned with the integration of circuits and systems onto small pieces of silicon today. A typical computer engineer has a working knowledge of silicon devices, CMOS circuits, logic design, and system architecture and is usually a specialist in one or more of these areas. The field of **electric and electronic engineering** has made spectacular advances in recent years. Overall, the objective has been to provide the technology needed to build large information systems on tiny chips, and to build a system of such chips in a board.

To solve modern computational problems new complex microprocessors are designed which due to their advanced architecture provide to solve these problems in real time and with minimized energy consumption, and with the speed of several billion instructions per second. The modern microprocessor consists of both microprocessor core and a set of peripheral devices including memory units, data transfer ports, DSP engines, ADC, DAC, etc. Such a microprocessor or a system of them is named as System-On-the-Chip, or shortly SOC.

The fact is that the modern SOC is characterized by high wire resistivity, high level of signal interferences, the delays in wires which are higher than delays in gates, low feeding voltage and high current consumtion. The frequencies in such circuits are higher than thousands of megahertz. Therefore, to design modern SOC the computer engineer has to take many electical laws into account.

He or she has to know that any board interconnection is a transmission line by definition. Therefore, to design modern computer boards one has to consider that the reflections, interference, and noise in board wires cause measurable changes in the appearance or behavior of signals at higher frequencies. And as a result, negleblible attention to these features causes unworkable projects.

Lectures in electronic engineering include electric engineering basics, transistor basics, transistor circuit design questions, and MOSFET transistor basics, which are needed in development of modern circuits. The lectures are based on the method of quadripole analysis. This method helps to investigate and design both transmission lines and transistor circuits. Besides, the quadripole method use shows the way to solve the complex problem of the circuit analysis by the approach "divide and concuer". This teach the students to develop the algorithms for solving similar problems.

Used literature

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Electric engineering basics

1. Basic electric circuits and components

1.1. Electric measurements

The electric **current** is the stream, or continuous movement of electric charges as illustrated by the fig. 1. The electric current is the time rate of change of charge across the referenced area, as given by:

$$i = \frac{dQ}{dt},$$

where Q is **charge**, which is measured in units of coulombs. A coulomb is equal to charge of approximately $6{,}24 \cdot 10^{18}$ electrons.

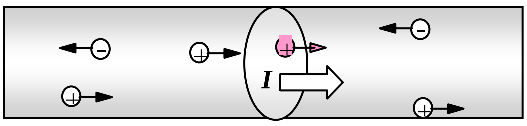


Fig. 1. Illustration of the electric current flow

The current unit is **ampere**. It is the base SI unit. SI means Systėme International d'Unitės. This is the internationally agreed upon system of coherent units that is now in use for all scientific and most technological purposes in many countries. In short, 1 ampere is 1A. Milliampere (mA), microampere (μA), nanoampere (nA) and other subunits are distinguished. These units depend on each other: 1A =1000 mA, 1mA =1000 μA, 1μA =1000 nA.

In the space the electric potential *e* is distinguished. It means the work, which is necessary to bring a unit of positive charge to a given point of the space. The difference between potentials in two points, say e_1, e_2 is equal to the voltage *v*, i.e.

$$v = e_1 - e_2.$$

The **voltage** *v* is the electromotive force, which is equal to the work, which is necessary to bring a unit of positive charge from one point to another. From this point of view the potential *e* is equal to the voltage between this point and some abstract point of zeroed potential e_0. Often the point with such properties is called as a ground.

The voltage, or **potential difference**, between two points in a circuit indicates the energy required to move charge from one point to the other. As will be presently shown, the direction, or polarity, of the voltage is closely tied to whether energy is being dissipated or generated in the process.

Both potential and voltage are measured in **volts**, shortly, V. Such subunits like kilovolt (kV), millivolt (mV), microvolt (μV) are frequently used, and 1kV=1000V; 1V=1000mV; 1mV=1000μV.

In the low-frequency electronic devices the **direct-current** (DC), and **alternating-current** (AC) circuits are distinguished. Below the current and voltage are considered which are exchanged in time, mostly alternating-current and alternating-voltage.

If for some short time period some charge *Q* was flown throug a subcircuit, i.e. the current *i* flows, and at its edges the voltage *v* is present. Then the elementary **energy** which was emitted there as warm, radio waves, chemical compound, etc, is equal to

$$dW = VdQ = vidt.$$

The speed of the energy income in the circuit is the instantaneous **power** and is equal to

$$P = \frac{dW}{dt} = \frac{vdQ}{dt} = vi\ .$$

The energy which is emitted from time t_1 to time t_2 is equal to

$$W = \int_{t_2}^{t_1} Pdt\ .$$

The energy and power are measured in **joules** (J) and **watts** (W). The power also is measured in kilowatts (kW), miliwatts (mW), 1 kW =1000 W; 1 mW = 0.001 W.

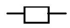
1.2.Resistance and Ohm law

Between voltage and current the following dependence is present named the **Ohm law**:

$$i = \frac{v}{R},$$

where v is the voltage between two given points; i is the current which is moved out one point and flowed in another point with less potential; R is the **resistance** between these points. In general, the resistance R in this equation is substituted by the **impedance** Z, which is more complex nature (see below).

The resistance is measured in **ohms** (Ω), kiloohms (kΩ), megaohms (MΩ), and 1 kΩ =1000 Ω; 1 MΩ = 1000 kΩ.

The electro engineering unit with the given resistance, named **resistor** is depicked as: .

For homogeneous conductors with the steady intersection area S (see Fig. 2) its resistance is equal to:

$$R = \frac{l}{\sigma S},$$

where σ is the conductor **conductivity**; l is its length.

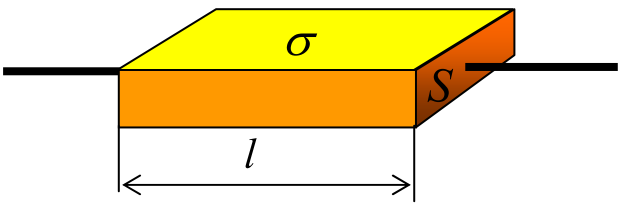


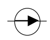
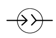
Fig 2 – Resistance of the metallic bar

The power dissipation P of a resistor is given by

$$P = vi = i^2 R. \qquad (2)$$

In computer engineering, this type of power dissipation leads to heating and can cause thermal instabilities and circuit failures. The formula (2) analysis shows that for the same voltage a small value of R gives large current flow, and is accompanied by large power dissipation.

1.3. Voltage and Current sources

In the electro engineering the **voltage sources** are signified as: , and the **current sources** are signified as: .

An ideal voltage source provides a prescribed voltage across its terminals irrespective of the current flowing through it. The amount of current supplied by the source is determined by the circuit connected to it. The ideal voltage source has zeroed inner resistance $r_s = 0$. And ideal current source has zeroed inner conductivity, i.e. unlimited inner resistance $r_s = \infty$.

Really, such a source can be accumulator, transducer, or generator output, which has concrete inner resistance $r_s > 0$. The symbol of such **voltage source** as accumulator, or alkaline battery is .

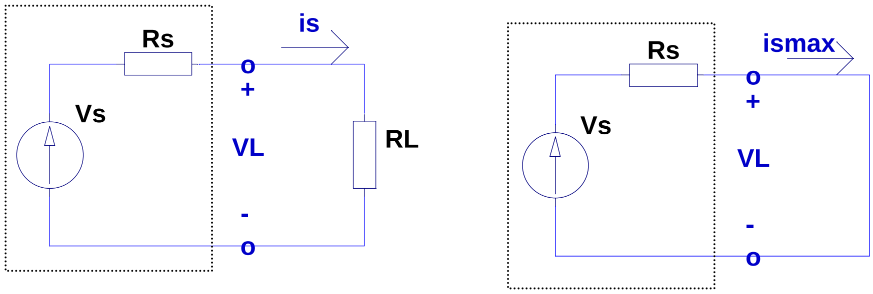


Fig. 3 – Practical voltage source under loading (a) and under shorting (b)

Figure 3 depicts a model for a practical voltage source, composed of an ideal voltage source, v_S, in series with a resistance, R_S. Note that by convention the **direction of positive current flow** out of a voltage source is *out of the positive terminal.*

The resistance R_S in effect poses a limit to the maximum current the voltage source can provide:

$$i_{S\max} = v_S / R_S$$

Note, however, that its presence affects the voltage across the load resistance: this voltage is no equal to the source voltage. Since the current provided by the source is

$$i_S = \frac{v_S}{R_S + R_L}$$

the load voltage can be determined to be

$$v_L = i_S R_L = v_S \frac{v_S}{R_S + R_L} .$$

The circuit in Fig. 3 suggests that the ideal voltage source is required to provide an infinite amount of current to the load, in the limit as the load resistance approaches zero. Naturally, this is impossible; for example, consider a conventional car battery: 12 V, 450 A-h (ampere-hours). This implies that there is a limit to the amount of current a practical source can deliver to a load. The limitations of practical sources can be approximated by exploiting the notion of the internal resistance of a source.

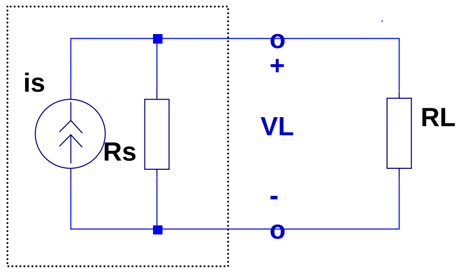


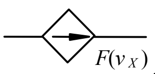
Fig. 4 – Practical current source under loading

A similar modification of the ideal current source model is useful to describe the behavior of a practical current source. The circuit illustrated in Figure 4 depicts a simple representation of a practical current source, consisting of an ideal source in parallel with a resistor. Note that as the load resistance approaches infinity (i.e., an open circuit), the output voltage of the current source approaches its limit,

$$v_{S\,\max} = i_S r_S .$$

A good current source should be able to approximate the behavior of an ideal current source. Therefore, a desirable characteristic for the internal resistance of a current source is that it be as large as possible.

The sources described so far have the capability of generating a prescribed voltage or current independent of any other element within the circuit. Thus, they are termed *independent sources.*

There exists another category of sources, however, whose output (current or voltage) is a function of some other voltage or current in a circuit. These are called **dependent** (or **controlled**) **sources**. A different symbol, in the shape of a diamond, is used to represent dependent sources and to distinguish them from independent sources. For example, voltage controlled current source (VCCS) : 

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1.4. Time–dependent signal sources

Figure 5 illustrates the convention that will be employed to denote time-dependent signal sources.

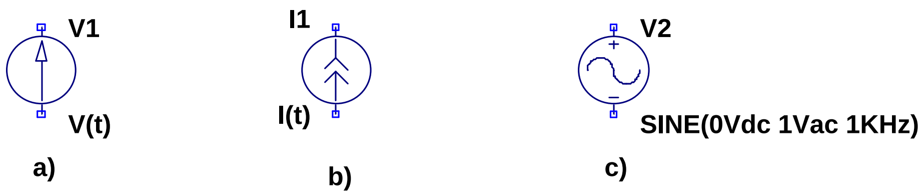


Fig. 5 – Generalized time-dependent sources (a,b), and sinusoidal source (c)

One of the most important classes of time-dependent signals is that of **periodic signals.** These signals appear frequently in practical applications and are a useful approximation of many physical phenomena. A periodic signal $x(t)$ is asignal that satisfies the following equation:

$x(t) = x(t + nT)\ n = 1, 2, 3, \ldots$

where T is the **period** of $x(t)$. Figure 6 illustrates a number of the periodic waveforms that are typically encountered in the study of electrical circuits.

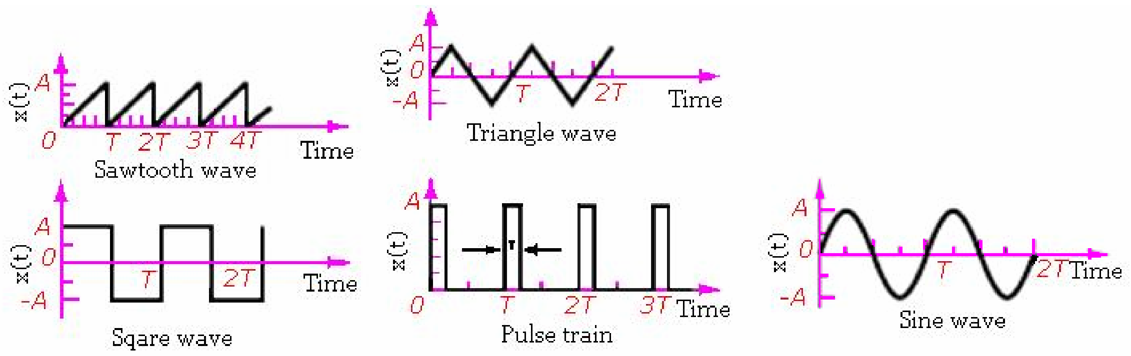


Fig. 6 – Typical waveforms of the time-dependent signal circuits

Waveforms such as the sine, triangle, square, pulse, and sawtooth waves are provided in the form of voltages (or, less frequently, currents) by commercially available **signal** (or **waveform**) **generators.** Such instruments allow for selection of the waveform peak amplitude, and of its period. Sinusoidal waveforms constitute by far the most important class of time-dependent signals. Figure 6 depicts the relevant parameters of a sinusoidal waveform. A generalized sinusoid is defined as follows:

$$x(t) = A\cos(\omega t + \varphi)$$

where A is the **amplitude,** ω the **radian frequency,** and φ the **phase.**

If f = natural frequency = $1/T$(cycles/s, or Hz)

Then ω = radian frequency = $2\pi f$ (radians/s)

The phase shift, φ, permits the representation of an arbitrary sinusoidal signal.

Thus, the choice of the reference cosine function to represent sinusoidal signals — arbitrary as it may appear at first—does not restrict the ability to represent all sinusoids.

1.5. Average and RMS Values

Now that a number of different signal waveforms have been defined, it is appropriate to define suitable measurements for quantifying the strength of a time-varying electrical signal. The most common types of measurements are the **average** (or **DC**) **value** of a signal waveform—which corresponds to just measuring the mean voltage or current over a period of time—and the **root-mean-square** (or **rms**) **value,** which takes into account the fluctuations of the signal about its average value. Formally, the operation of computing the average value of a signal corresponds to integrating the signalwaveform over some (presumably, suitably chosen) period of time. We define the time-averaged value of a signal *x(t)* as

$$\langle x(t)\rangle = \frac{1}{T}\int_0^T x(t)dt,$$

where T is the period of integration. Figure 7 illustrates how this process does. In fact, it corresponds to computing the average amplitude of *x(t)* over a period of *T* seconds.

Fig. 7 – Time-averaged value of the signal *x(t)*

Problem

Compute the average value of the signal $x(t) = 10\cos(100t)$.

Analysis: The signal is periodic with period $T = 2\pi/\omega = 2\pi/100$, thus we need to integrate over only one period to compute the average value:

$$\langle x(t)\rangle = \frac{1}{T}\int_0^T x(t')dt' = \frac{100}{2\pi}\int_0^{2\pi/100} 10\cos(100t)dt = \frac{10}{2\pi}\langle \sin(2\pi) - \sin(0)\rangle = 0$$

Comments: The average value of a sinusoidal signal is zero, independent of its amplitude and frequency.

Very conveniently, a useful measure of the voltage of an AC waveform is the root-mean-square, or **rms**, value of the signal, *x(t)*, defined as follows:

$$x_{rms} = \sqrt{\frac{1}{T}\int_0^T x^2(t')dt'}$$

Note that if *x(t)* is a voltage, the resulting x_{rms} will also have units of volts. If you analyze equation 4.24, you can see that, in effect, the rms value consists of the square *r*oot of the average (or *m*ean) of the *s*quare of the signal.

Thus, the notation *rms* indicates exactly the operations performed on *x(t)* in order to obtain its rms value.

Problem

Compute the rms value of the sinusoidal current $i(t) = I\cos(\omega t)$.

Analysis: Applying the definition of rms value we compute:

$$i_{rms} = \sqrt{\frac{1}{T}\int_0^T i^2(t')dt'} = \sqrt{\frac{\omega}{2\pi}\int_0^{2\pi/\omega} I^2\cos^2(\omega\, t')dt'} = \sqrt{\frac{\omega}{2\pi}\int_0^{2\pi/\omega} I^2\left(\frac{1}{2}+\cos(2\omega\, t')\right)dt'} =$$

$$= \sqrt{\frac{1}{2}I^2 + \frac{\omega}{2\pi}\int_0^{2\pi/\omega}\frac{I^2}{2}\cos(2\omega\, t')dt'} = \sqrt{\frac{1}{2}I^2} = \frac{I}{\sqrt{2}} = 0{,}707I$$

where I is the **peak value** of the waveform *i(t)*.

Comments: The rms value of a sinusoidal signal is equal to 0.707 times the peak value, independent of its amplitude and frequency. The factor of $0.707 = 1/\sqrt{2}$ is a useful number to remember, since it applies to any sinusoidal signal.

1.6. Phasors and impedance

In this section, we introduce an efficient notation to make it possible to represent sinusoidal signals as *complex numbers,* and to eliminate the need for solving differential equations.

Named after the Swiss mathematician Leonhard Euler, **Euler's identity** forms the basis of phasor notation. Simply stated, the identity defines the **complex exponential** $e^{j\theta}$ as a point in the complex plane, which may be represented by real and imaginary components:

$$e^{j\theta} = \cos\theta + j\sin\theta\,.$$

Figure 8 illustrates how the complex exponential may be visualized as a point (or vector, if referenced to the origin) in the complex plane.

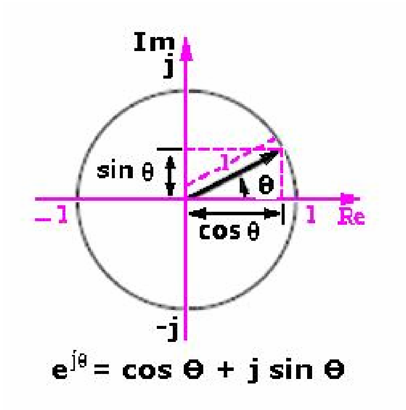


Fig. 8 – Complex data in the complex plane

Note that the magnitude of $e^{j\theta}$ is equal to 1: $|e^{j\theta}| = 1$ since

$$|\cos\theta + j\sin\theta| = \sqrt{(\cos^2\theta + \sin^2\theta)} = 1$$

Note also that the Euler's identity corresponds to equating the polar form of a complex number to its rectangular form. For example, consider a vector of length A making an angle θ with the real axis. The following equation illustrates the relationship between the rectangular and polar forms:

$$Aej\theta = A\cos\theta + jA\sin\theta = A\angle\theta$$

To see how complex numbers can be used to represent sinusoidal signals, rewrite the expression for a generalized sinusoid in light of Euler's equation:

$$A\cos(\omega t{=}\varphi){=}Re[Ae^{j(\omega t+\varphi)}]$$

We see, that *it is possible to express a generalized sinusoid as the real part of a complex vector* whose **argument,** or **angle,** is given by $(\omega t + \varphi)$ and whose length, or **magnitude,** is equal to the peak amplitude of the sinusoid. The **complex phasor** corresponding to the sinusoidal signal $A\cos(\omega t + \varphi)$ is therefore defined to be the complex number

$Ae^{j\varphi}$ = complex phasor notation for $A\cos(\omega t+\varphi)= A\angle\theta$

Problem

Compute the phasor voltage resulting from the series connection of two sinusoidal voltage sources $v1(t) = 15\cos(314t + \pi/4)$ V

$v2(t) = 15\cos(314t + \pi/12)$ V

Find: Equivalent phasor voltage $vS(t)$.

Analysis: Write the two voltages in phasor form:

$\mathbf{V}_1(j\omega) = 15\angle\pi/4$ V

$\mathbf{V}_2(j\omega) = 15e^{j\pi/12} = 15\angle\pi/12$ V

Convert the phasor voltages from polar to rectangular form:

$\boldsymbol{V}_1(j\omega) = 10.61 + j10.61$ V

$\boldsymbol{V}_2(j\omega) = 14.49 + j3.88$

Then

$\boldsymbol{V}_S(j\omega) = \boldsymbol{V}_1(j\omega) + \boldsymbol{V}_2(j\omega) = 25.10 + j14.49 = 28.98\, e^{j\pi/6} = 28.98 \angle \pi/6$ V

Now we can convert $\boldsymbol{V}_S(j\omega)$ to its time-domain form:

$v_S(t) = 28.98 \cos(314t + \pi/6)$ V.

Phasor notation is a very efficient technique to solve AC circuit problems.

1.7. Impedance

We now analyze the *i*-*v* relationship of the three ideal circuit elements in light of the phasor notation. The result will be a new formulation in which not only resistor but any linear two pole circuits will be described in this notation. A direct consequence of this result will be that the Ohm law and a set of theorems are extended to AC circuits. In the context of AC circuits, any one of the ideal linear circuit elements will be described by a parameter called **impedance,** which may be viewed as a **complex resistance.**

Figure 9 depicts the circuit represented in phasor-impedance form; the latter representation explicitly shows phasor voltages and currents and treats the circuit element as a generalized "impedance."

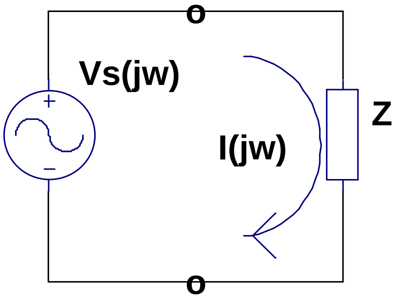


Fig. 9 – Simple circuit represented in phasor-impedance form

It will be shown that each of the ideal circuit elements may be represented by one such impedance element. Let the source voltage in the circuit of Figure be defined by

$$v_S(t) = A\cos \omega t \quad \text{or} \quad \boldsymbol{V}_S(j\omega) = Ae^{j0^\circ} = A \angle 0^\circ.$$

Then the current $i(t)$ is defined by the *i*-*v* relationship for each circuit element. Let us examine the properties of the resistor, inductor, and capacitor, which are the general elements of any circuit.

1.8. Capacitance

The **capacitance** is a parameter that describes how a particular device can store electric charge. The **capacitor** is formed by two metal plates that are separated by an insulator layer. The circuit symbol for a capacitor is: ⊣⊢. If a voltage v is applied to the plates, then charges Q of opposite signs will be induced on these plates. The amount of charge Q stored on the capacitor and the applied voltage v are in the following relation

$$Q = Cv,$$

where C is capacitance. The unit of capacitance is the **farad** (F). In the world of electronics, realistic capacitors have very small values and are measured in microfarads (μF), nanofarads (nF), and picofarads (pF), and 1 μF=10^{-6}F; 1nF=10^{-9}F; 1pF=10^{-12}F.

When moving charge from or to the capacitor then a current I flows with the value

$$I = \frac{dQ}{dt} = C\frac{dv}{dt},$$

that is proportional to the time rate of change the voltage.

The electric field energy in the capacitor in arbitrary time is equal to

$$W = \frac{Cv^2}{2} = \frac{Q^2}{2C}. \qquad (3)$$

The formulae (3) shows that the capacitance can store the energy. But this energy in most cases is the alternating one, and could not be stored more than for seconds or minutes. The capacitors in the modern dynamic random access memories (DRAMs) are used for the data storing. The positive or negative charge in them means the bit, which is equal to 1 or 0. To store the bits for a long time these storage capacitors have to be uploaded (refreshed) automatically in the time period of some milliseconds.

With $i_C = i$ and $v_C = v_S$, the capacitor current may be expressed as:

$$i_C(t) = C\frac{dv_C(t)}{dt} = C\frac{d}{dt}\big(A\cos(\omega\ t)\big) = -C\big(A\omega\sin(\omega\ t)\big) = \omega CA\cos\big(\omega\ t + \pi/2\big)$$

so that, in phasor form,

$\boldsymbol{V}_S\ (j\omega) = A\angle 0$, and $\boldsymbol{I}(j\omega) = \omega CA \angle \pi/2$

The impedance of the ideal capacitor, $Z_C(j\omega)$, is therefore defined as follows:

$$Z_C(j\omega) = \frac{V_S(j\omega)}{I(j\omega)} = \frac{1}{\omega C}\angle -\pi/2 = \frac{-j}{\omega C} = \frac{1}{j\omega C}$$

where we have used the fact that $1/j = e^{-j\pi/2} = -j$. Thus, the impedance of a capacitor is also a frequency-dependent complex quantity, with the impedance of the capacitor varying as an inverse function of frequency; and so a capacitor acts like a short circuit at high frequencies, whereas it behaves more like an open circuit at low frequencies.

1.9. Inductance

The **inductance** *L* is the entity of the electric net which has the properties of the inductive coil in which the magnetic energy can be loaded. If the voltage at the ends of the inductivity L is equal to

$$V = L\frac{di}{dt} = \frac{d\Psi}{dt}, \tag{4}$$

where Ψ is the magnetic **linkage** in the inductance. For the inductance formed by the coil with *w* windings the magnetic linkage is $\Psi = w\Phi$, where Φ is the magnetic **flux**. The magnetic linkage and flux are measured in **webers** (Wb). The circuit symbol for an inductance is: .

An inductance of one **henry**, abbreviated H, represents a potential difference of one volt across an inductor within which the current is increasing or decreasing at one ampere per second. Usually, inductances are expressed in millihenries (mH), microhenries (μH), or even in nanohenries (nH). Then 1 mH = 0.001 H; 1 μH =0.001mH; and 1 nH = 0.001μH.

The magnetic energy in the inductance is equal to

$$W = \frac{Li^2}{2} = \frac{\Psi^2}{2L}. \tag{5}$$

The inductances are widely used in the modern switching AC-DC, and DC-DC converters, which serve as the voltage sources of the computers. The inductances are used in the computer circuits for the current filtering as well. The formulas (4), (5) show, that the inductance can load the high energy, and the voltage in it can have high figures when the current is high, and it is exchanged sharply. Therefore, care have to be taken to keep the current exchange in the inductances of such circuits to save the integral circuits from the dramatic voltage surges.

A set of coils, which have the common magnetic flux is named as a **transformer**. In the transformer the alternating current in the primary coil induces the alternating magnetic flux, which generates the alternating voltage, named **electromotive force** (EMF) in the secondary coil. The transformer steps up or steps down the input voltage depending on the rate of the secondary and primary windings.

Let $v_L(t) = v_S(t)$ and $i_L(t) = i(t)$

Then the following expression may be derived for the inductor current:

$$i_L(t) = i(t) = \frac{1}{L}\int v_S(t')dt' = i_L(t) = \frac{1}{L}\int A\cos(\omega\ t')dt' = \frac{A}{\omega L}\sin(\omega\ t)$$

Note how a dependence on the radian frequency of the source is clearly present in the expression for the inductor current. Further, the inductor current is shifted in phase (by 90◦) with respect to the voltage. This fact can be seen by writing the inductor voltage and current in time-domain form:

$$v_S(t) = v_L(t) = A\cos(\omega\ t)$$

$$i(t) = i_L(t) = \frac{A}{\omega\ L}\cos\left(\omega\ t - \frac{\pi}{2}\right)$$

It is evident that the current is not just a scaled version of the source voltage, as it was for the resistor. Its magnitude depends on the frequency, *ω*, and it is shifted (delayed) in phase by *π/*2 radians, or 90◦. Using phasor notation, equation becomes

$$V_S(j\omega) = A\angle 0$$

$$I(j\omega) = \frac{A}{\omega L}\angle \pi / 2$$

Thus, the impedance of the inductor is defined as follows:

$$Z_L(j\omega) = \frac{V_S(j\omega)}{I(j\omega)} = \omega L\angle \pi / 2 = j\omega L$$

Note that the inductor now appears to behave like a *complex frequency-dependent resistor,* and that the magnitude of this complex resistor, *ωL*, is proportional to the signal frequency, *ω*. Thus, an inductor will “impede” current flow in proportion to the sinusoidal frequency of the source signal. This means that at low signal frequencies, an inductor acts somewhat like a short circuit, while at high frequencies it tends to behave more as an open circuit.

1.10. Impedance meanings

The impedance parameter is extremely useful in solving AC circuit analysis problems, because it will make it possible to take advantage of most of the network theorems developed for DC circuits by replacing resistances with complex-valued impedances. The examples, that follow, illustrate how branches containing series and parallel elements may be reduced to a single equivalent impedance. It is important to emphasize that although the impedance of simple circuit elements is either purely real (for resistors) or purely imaginary (for capacitors and inductors), the general definition of impedance for an arbitrary circuit must allow for the possibility of having both a real and an imaginary part, since practical circuits are made up of more or less complex interconnections of different circuit elements.

In its most general form, the impedance of a circuit element is defined as the sum of a real part and an imaginary part:

$$Z(j\omega) = R(j\omega) + jX(j\omega)$$

where R is called the **AC resistance** and X is called the **reactance.** The frequency dependence of R and X has been indicated explicitly, since it is possible for a circuit to have a frequency-dependent resistance. Note that the reactances have units of ohms, and that **inductive reactance** is always positive, while **capacitive reactance** is always negative.

Figure 10 depicts $Z_C(j\omega)$ in the complex plane, alongside $Z_R(j\omega)$ and $Z_L(j\omega)$.

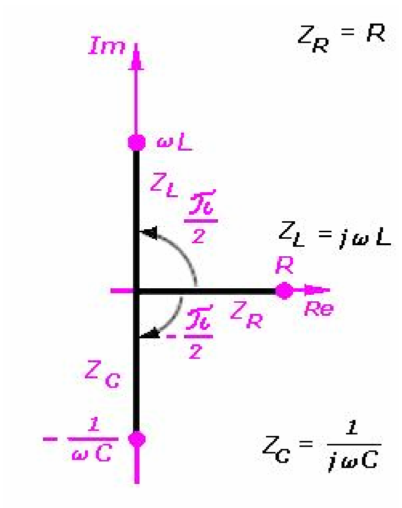
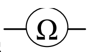


Fig. 10 – Impedance of resistor, capacitor, and inductor in the complex plane

1.11. Measuring devices

The **ohmmeter** is a device that, when connected across a circuit element, can measure the resistance of the element. Figure 11 depicts the circuit connection of an ohmmeter to a resistor. Symbol for ohmmeter is  .

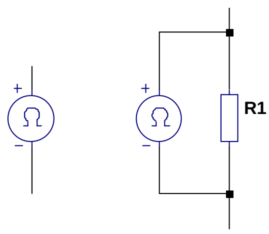
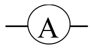


Fig. 11 – Connection of the ohmmeter

One important rule needs to be remembered: The resistance of an element can be measured only when the element is disconnected from any other circuit.

The **ammeter** is a device that, when connected in series with a circuit element, can measure the current flowing through the element Symbol for ammeter is 

. Figure 12 depicts the connection of an ohmmeter into a circuit.

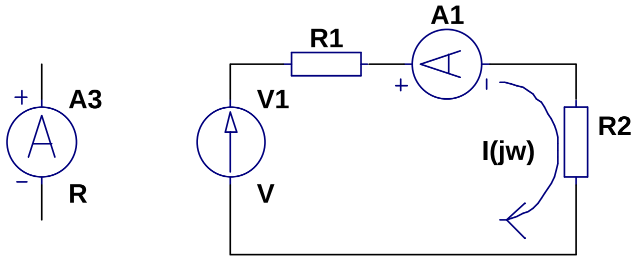
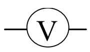


Fig. 12 – Connection of an ammeter into a circuit

The ammeter must be placed in series with the element whose current is to be measured (e.g., resistor $R1$ or $R2$). The ammeter should not restrict the flow of current (i.e., cause a voltage drop), or else it will not be measuring the true current flowing in the circuit. An ideal ammeter has zero internal resistance.

The **voltmeter** is a device that can measure the voltage across a circuit element (see Fig. 13). Since voltage is the difference in potential between two points in a circuit, the voltmeter needs to be connected across the element whose voltage we wish to measure. The symbol for voltmeter is 

. A voltmeter must also fulfill the following requirements. The voltmeter must be placed in parallel with the element whose voltage it is measuring. The voltmeter should draw no current away from the element whose voltage it is measuring, or else it will not be measuring the true voltage across that element. Thus, an ideal voltmeter has infinite internal resistance.

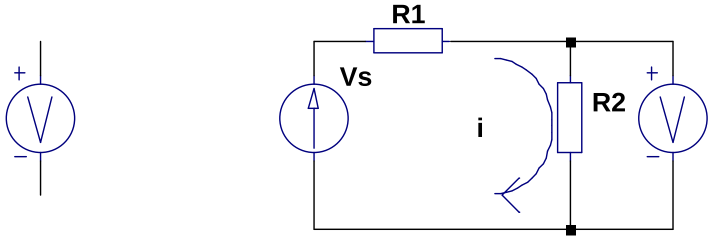


Fig. 13 – Connection of a voltmeter into a circuit

All of the considerations that pertain to practical ammeters and voltmeters can be applied to the operation of a **wattmeter**, a measuring instrument that provides a measurement of the power dissipated by a circuit element, since the wattmeter is in effect made up of a combination of a voltmeter and an ammeter. Symbol for the wattmeter is  .

Figure 14 depicts the typical connection of a wattmeter in the same series circuit used above. In effect, the wattmeter measures the current flowing through the load and, simultaneously, the voltage across it and multiplies the two to provide a reading of the power dissipated by the load.

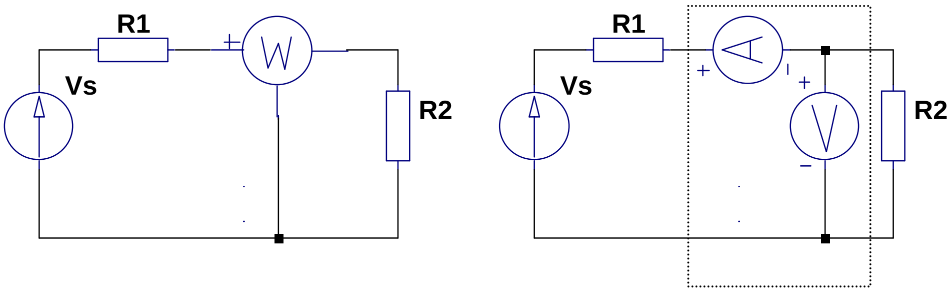


Fig. 14 – Connection of a wattmeter into a circuit

1.12. Linear and unlinear components and circuits

The relationship between current and voltage at the terminals of a circuit element defines the behavior of that element within the circuit. In this section we shall introduce a graphical means of representing the terminal characteristics of circuit elements.

Suppose now that a known voltage were imposed across a circuit element. The current that would flow as a consequence of this voltage, and the voltage itself, form a unique pair of values. If the voltage applied to the element were variedand the resulting current measured, it would be possible to construct a functional relationship between voltage and current known as the ***i-v* characteristic** (or **voltampere characteristic**). Such a relationship defines the circuit element, in the sense that if we impose any prescribed voltage (or current), the resulting current (or voltage) is directly obtainable from the *i-v* characteristic. A direct consequence is that the power dissipated (or generated) by the element may also be determined from the *i-v* curve.

Figure 15 depicts the *i-v* characteristic of a tungsten filament light bulb. A variable voltage source is used to apply various voltages, and the current flowing through the element is measured for each applied voltage.

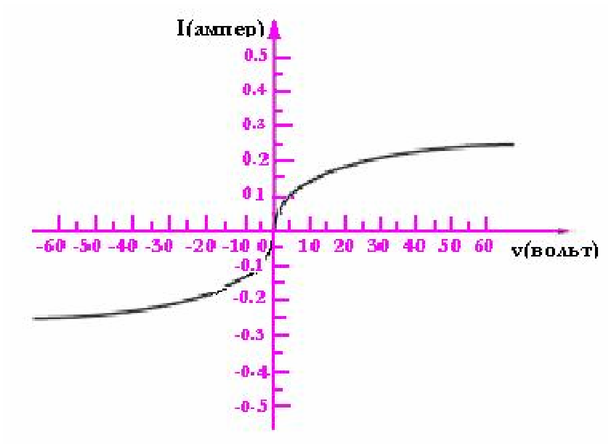


Fig 15 – *i-v* characteristic of a tungsten filament light bulb

We could certainly express the *i-v* characteristic of a circuit element in functional form:

$i = f(v)$ $v = g(i)$.

The examples of the unlinear components are varistor (voltage dependent resistor), diode, coil inductance. More complex components are transistors, coil transformers, triacs.

Due to the presence of unlinear components in the electrical circuit the linear and unlinear circuits are distinguished. In general, any real circuit can be considered as unlinear one because any component is not ideal one.

The analysis and synthesis of the unlinear circuits are much complex of that of linear circuits. Therefore, to deal with the unlinear circuits its unlinear components are usually linearized, and the circuit is considered at its state where the components are represented as linear ones.

2. DC electrical circuits and networks

2.1. Electrical circuit elements

In the previous sections we have outlined models for the basic circuit elements: sources, resistors, capacitors, inductances and measuring instruments. In order for current to flow there must exist a closed circuit. We have assembled all the tools and parts we need in order to define an **electrical network**. It is appropriate to formally define the elements of the electrical circuit; the definitions that follow are part of standard electrical engineering terminology.

A **branch** is any portion of a circuit with two terminals connected to it. A branch may consist of one or more circuit elements. In practice, any circuit element with two terminals connected to it is a branch.

A **node** is the junction of two or more branches (one often refers to the junction of only two branches as a *trivial node*). Figure 16 illustrates the concept. In effect, any connection that can be accomplished by soldering various terminals together is a node. It is very important to identify nodes properly in the analysis of electrical networks.

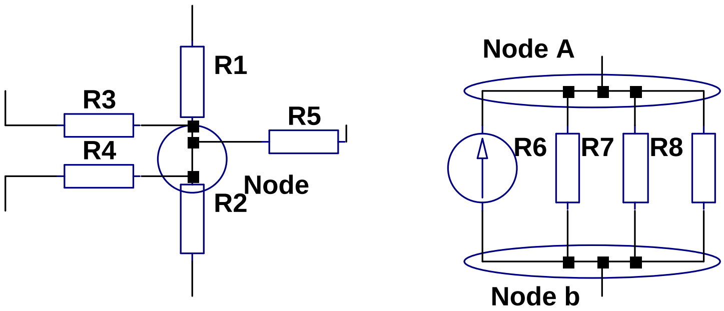


Fig. 16 – Node of the electrical circuit

A **loop** is any closed connection of branches. Various loop configurations are illustrated in Figure 17.

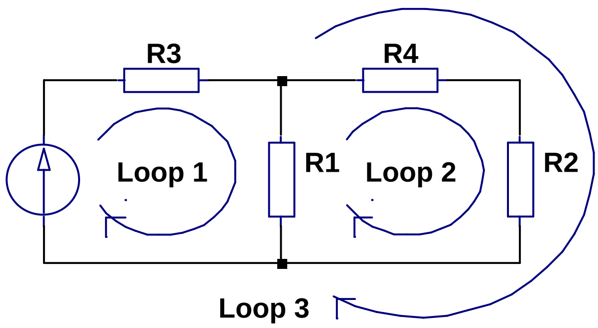
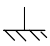


Fig. 17 – Loops in the circuit

A **mesh** is a loop that does not contain other loops. Meshes are an important aid to certain analysis methods. In Figure 17, the circuit consists of two meshes: loops 1 and 2 are meshes, but loop 3 is not a mesh, because it encircles both loops 1 and 2.

Whenever we reference the voltage at a node in a circuit, we imply an assumption that the voltage at that node is the potential difference between the node itself and a reference node called **ground**, which is located somewhere else in the circuit and which for convenience has been assigned a potential of zero volts.

The choice of the word *ground* is not arbitrary. In every circuit a point can be defined that is recognized as "ground" and is assigned the electric potential of zero volts for convenience. Symbol for ground is ⊥ , or ⏚, or  depending on some properties of the ground wire. For example, the first symbol is used as the digital signal ground, and the latter one as the analog signal ground is.

2.3. Kirchhoff's current law

Note that in the circuit of Figure 3, a the current, *i*, flowing from the voltage source to the resistor is equal to the current flowing from the resistor to the source. In other words, no current (and therefore no charge) is "lost" around the closed circuit. This principle was observed by the German scientist G. R. Kirchhoff and is now known as **Kirchhoff's current law (KCL).** Kirchhoff's current law states that because charge cannot be created but must be conserved, *the sum of the currents at a node must equal zero*. Formally:

$$\sum_{n=1}^{N} i_n = 0$$

The significance of Kirchhoff's current law is illustrated in Figure 18, where the simple circuit of Figure 3,a has been augmented by the addition of two resistors. In applying KCL, one usually defines currents entering a node as being negative and currents exiting the node as being positive. Thus, the resulting expression for node 1 of the circuit of Figure 18 is:

$$-i + i_1 + i_2 + i_3 = 0$$

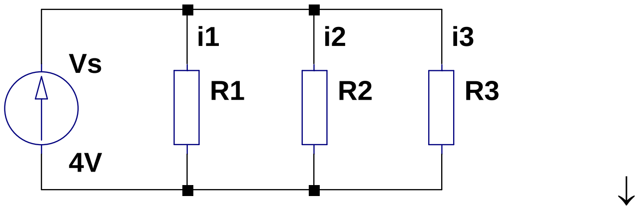


Fig. 18 – Kirhoff's law representation

Kirchhoff's current law is one of the fundamental laws of circuit analysis, making it possible to express currents in a circuit in terms of each other; for example, one can express the current leaving a node in terms of all the other currents at the node. The ability to write such equations is a great aid in thesystematic solution of large electric circuits.

Problem:

If the battery in the diagram supplies a total of 10mW to the three elements shown and $i_1 = 2$ mA and $i_2 = 1.5$ mA, what is the current i_3? If $i1 = 1$ mA and $i_3 = 1.5$ mA, what is i_2?

2.4. Kirchhoff's voltage law

The voltage, or **potential difference**, between two points in a circuit indicates the energy required to move charge from one point to the other. The principle underlying **Kirchhoff's voltage law** is that no energy is lost or created in an electric circuit; in circuit terms, the sum of all voltages associated with sources must equal the sum of the load voltages, so that *the net voltage around a* *closed circuit is zero.* If this were not the case, we would need to find a physical explanation for the excess (or missing) energy not accounted for in the voltages around a circuit. Kirchhoff's voltage law may be stated in a form similar to that used for **Kirchhoff's current law**:

$$\sum_{n=1}^{N} v_n = 0$$

where the *vn* are the individual voltages around the closed circuit.

Problem Known Quantities: Voltages across each circuit element; current in circuit.

Find: Power dissipated or generated by each element.

Analysis: Following the passive sign convention, we first select an arbitrary direction for the current in the circuit; the example will be repeated for both possible directions of current flow to demonstrate that the methodology is sound.

1. Assume clockwise direction of current flow, as shown in Figure 19.

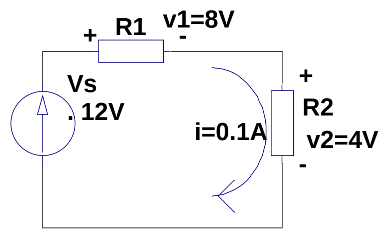


Figure 19 – Simple circuit to analyze

2. Label polarity of voltage source, as shown in Figure 19; since the arbitrarily chosen direction of the current is consistent with the true polarity of the voltage source, the source voltage will be a positive quantity.

3. Assign polarity to each passive element, as shown in Figure 19.

4. Compute the power dissipated by each element: Since current flows from − to + through the battery, the power dissipated by this element will be a negative quantity:

$P_B = -v_B * i = -(12\ \text{V}) * (0.1\ \text{A}) = -1.2\ \text{W}$

that is, the battery *generates* 1.2 W. The power dissipated by the two loads will be a positive quantity in both cases, since current flows from + to −:

$P_1 = v_1 * i = (8\ \text{V}) *. (0.1\ \text{A}) = 0.8\ \text{W}$

$P_2 = v_2 * i = (4\ \text{V}) *. (0.1\ \text{A}) = 0.4\ \text{W}$

2.5. Example: The Wheatstone Bridge

The **Wheatstone bridge** is a resistive circuit that is frequently encountered in a variety of measurement circuits. The general form of the bridge circuit is shown in Figure 20, where R_1, R_2, and R_3 are known while Rx is an unknown resistance, to be determined. The objective is to determine the unknown resistance, Rx .

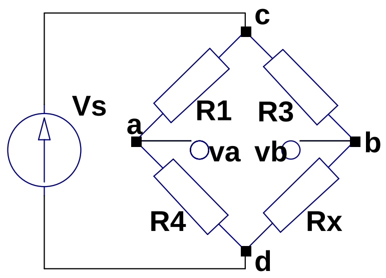


Fig. 20 – Wheatstone bridge circuit

1. Find the value of the voltage $v_{ab} = v_{ad} - v_{bd}$ in terms of the four resistances and the source voltage, v_S . Note that since the reference point d is the same for both voltages, we can also write $v_{ab} = v_a - v_b$.

2. If $R_1 = R_2 = R_3 =$ 1k″, v_S = 12 V, and v_{ab} = 12 mV, what is the value of Rx?

Solution

1. First, we observe that the circuit consists of the parallel combination of three subcircuits: the voltage source, the series combination of R_1 and R_2, and the series combination of R_3 and Rx . Since these subcircuits are in parallel, the same voltage will appear across each of them, namely, the source voltage, v_S .

Thus, the source voltage divides between each resistor pair, $R_1 - R_2$ and $R_3 - Rx$, according to the voltage divider rule: v_a is the fraction of the source voltage appearing across R_2, while vb is the voltage appearing across Rx :

$$v_a = v_S \frac{R_2}{R_1 + R_2} \qquad \text{and} \qquad v_b = v_S \frac{R_x}{R_3 + R_x}$$

Finally, the voltage difference between points a and b is given by:

$$v_{ab} = v_a - v_b = v_S \left(\frac{R_2}{R_1 + R_2} - \frac{R_x}{R_3 + R_x} \right)$$

This result is very useful and quite general.

2. In order to solve for the unknown resistance, we substitute the numerical values in the preceding equation to obtain $0{,}012 = 12\left(\frac{1{,}000}{2{,}000} - \frac{R_x}{1{,}000 + R_x}\right)$

2.6. Network analysis

The analysis of an electrical network consists of determining each of the unknown branch currents and node voltages. It is therefore important to define all relevant variables as clearly as possible, and in systematic fashion. Once the known and unknown variables have been identified, a set of equations relating these variables is constructed, and these are solved by means of suitable techniques. The analysis of electrical circuits consists of writing the smallest set of equations sufficient to solve for all of the unknown variables. The analysis of electrical circuits is greatly simplified if some standard conventions are followed.

The first observation to be made is that the relevant variables in network analysis are the node voltages and the branch currents. This fact is a consequence of Ohm's law. Consider the branch depicted in Figure 21, consisting of a single resistor.

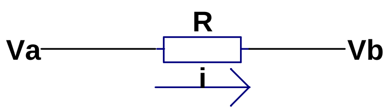


Fig. 21 – A single branch of a circuit

Here, once a voltage v_R is defined across the resistor R, a current i will flow through the resistor, according to $v_R = iR$. But the voltage v_R, which causes the current to flow, is really the difference in electric potential between nodes a and b:

$$v_R = v_a - v_b$$

Consider the circuit on the Fig. 22. Let us identify the branch and node voltages and the loop and mesh currents in the circuit.

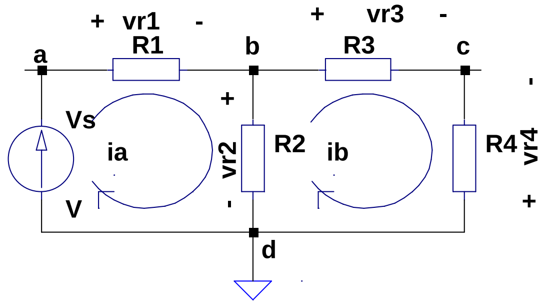


Fig. 22 – Analyzed circuit

The following node voltages may be identified:

Node voltages	Branch voltages
$v_a = v_S$ (sourse voltage)	$v_S = v_a - v_d = v_a$
$v_b = v_{R2}$	$v_{R1} = v_a - v_b$
$v_c = v_{R4}$	$v_{R2} = v_b - v_d = v_b$
$v_d = 0$ (ground)	$v_{R3} = v_b - v_c$
	$v_{R4} = v_c - v_d = v_c$

2.7. Node voltage method

The **node voltage method** is based on defining the voltage at each node as an independent variable. One of the nodes is selected as a **reference node** (usually—ground), and each of the other node voltages is referenced to this node. Once each node voltage is defined, Ohm's law may be applied between any two adjacent nodes in order to determine the current flowing in each branch. In the node voltage method, *each branch current is expressed in terms of one or more node voltages*; thus, currents do not explicitly enter into the equations.

Once each branch current is defined in terms of the node voltages, Kirchhoff's current law is applied at each node: $\Sigma i = 0$

The systematic application of this method to a circuit with n nodes would lead to writing n linear equations. However, one of the node voltages is the reference voltage and is therefore already known, since it is usually assumed to be zero. Thus, we can write $n-1$ *independent linear equations* in the $n-1$ independent variables (the node voltages). Nodal analysis provides the minimum number of equations required to solve the circuit, since any branch voltage or current may be determined from knowledge of nodal voltages.

The nodal analysis method may also be defined as a sequence of steps, as outlined below:

Node Voltage Analysis Method

1. Select a reference node (usually ground). All other node voltages will be referenced to this node.

2. Define the remaining $n-1$ node voltages as the independent variables.

3. Apply Kirhhoff Current Law at each of the $n-1$ nodes, expressing each current in terms of the adjacent node voltages.

4. Solve the linear system of $n-1$ equations in $n-1$ unknowns.

As an illustration of the method, consider the circuit shown in Figure 23.

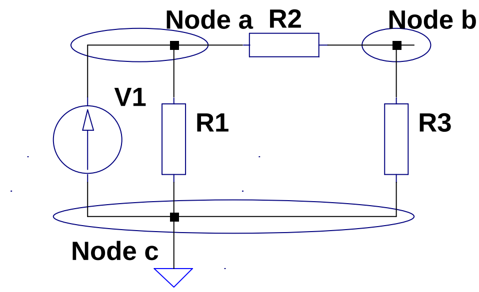


Fig. 23 – Example of the circuit solved by the Node Voltage Analysis Method

The direction of current flows selected arbitrarily (assuming that i_S is a positive current). Application of KCL at node a yields:

$i_S - i_1 - i_2 = 0$

whereas, at node b,

$i_2 - i_3 = 0$

It is instructive to verify that it is not necessary to apply KCL at the reference node. The equation obtained at node c,

$i_1 - i_3 - i_S = 0$.

is not independent of both previous equations. Now, in applying the node voltage method, the currents i_1, i_2, and i_3 are expressed as functions of v_a, v_b, and v_c, the independent variables. Ohm's law requires that i_1, for example, be given by

$i_1 = (v_a - v_c)/R_1$

since it is the potential difference, $v_a - v_c$, across R_1 that causes the current i_1 to flow from node a to node c. Similarly,

$i_2 = (v_a - v_b)/R_2$

$i_3 = (v_b - v_c)/R_3$

The presence of voltage sources actually simplifies the calculations. To illustrate this point, consider the circuit of Figure 24. Note that one of the node voltages is known already.

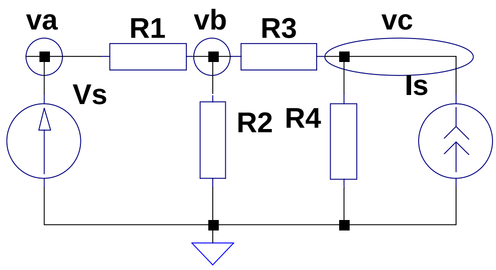


Fig. 24 – Circuit to analyze by the node voltage method

2.8 Mesh current method

The second method of circuit analysis, which is in many respects analogous to the method of node voltages, employs **mesh currents** as the independent variables. The idea is to write the appropriate number of independent equations, using mesh currents as the independent variables. Analysis by mesh currents consists of defining the currents around the individual meshes as the independent variables. Subsequent application of Kirchhoff's voltage law around each mesh provides the desired system of equations.

Consider the circuit on the Fig. 25. Let us identify the branch and node voltages and the loop and mesh currents in the circuit.

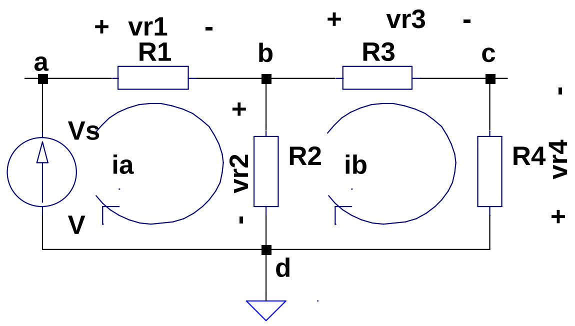


Fig. 25 – Analyzed circuit

In the mesh current method a current flowing through a resistor in a specified direction defines the polarity of the voltage across the resistor, as illustrated in Figure 21, and that the sum of the voltages around a closed circuit must equal zero, by Kirhhoff voltage law. Once a convention is established regarding the direction of current flow around a mesh, simple application of Kirhhoff voltage law provides the desired equation. Figure 24 illustrates this point.

The number of equations is equal to the number of meshes in the circuit. All branch currents and voltages may subsequently be obtained from the mesh currents, as will presently be shown. Since meshes are easily identified in a circuit, this method provides a very efficient and systematic procedure for the analysis of electrical circuits. The following box outlines the description of the procedure used in applying the mesh current method to a linear circuit.

Mesh Current Analysis Method

1. Define each mesh current consistently. We shall always define mesh currents clockwise, for convenience.

2. Apply Kirhhoff voltage law around each mesh, expressing each voltage in terms of one or more mesh currents.

3. Solve the resulting linear system of equations with mesh currents as the independent variables.

In mesh analysis, it is important to be consistent in choosing the direction of current flow. To avoid confusion in writing the circuit equations, mesh currents will be defined exclusively clockwise when we are using this method.

To illustrate the mesh current method, consider the simple two-mesh circuit shown in Figure 24. This circuit will be used to generate two equations in the two unknowns, the mesh currents i_1 and i_2. It is instructive to first consider each mesh by itself.

Beginning with mesh 1, note that the voltages around the mesh have been assigned in Figure 24 according to the direction of the mesh current, i_1. Recall that as long as signs are assigned consistently, an arbitrary direction may be assumed for any current in a circuit; if the resulting numerical answer for the current is negative, then the chosen reference direction is opposite to the direction of actual current flow. Thus, one need not be concerned about the actual direction of current flow in mesh analysis, once the directions of the mesh currents have been assigned.

According to the sign convention, then, the voltages v_1 and v_2 are defined as shown in Figure 24. Now, it is important to observe that while mesh current i_1 is equal to the current flowing through resistor R_1 (and is therefore also the branch current through R_1), it is not equal to the current through R_2. The branch current through R_2 is the difference between the two mesh currents, $i_1 - i_2$. Thus, since the polarity of the voltage v_2 has already been assigned, according to the convention, it follows that the voltage v_2 is given by:

$$v_2 = (i_1 - i_2)R_2$$

Finally, the complete expression for mesh 1 is

$$v_S - i_1R_1 - (i_1 - i_2)R_2 = 0$$

The mesh current i_2 is also the branch current through resistors R_3 and R_4; however, the current through the resistor that is shared by the two meshes, R_2, is now equal to $(i_2 - i_1)$, and the voltage across this resistor is

$$v_2 = (i_2 - i_1)R_2$$

and the complete expression for mesh 2 is

$$(i_2 - i_1)R_2 + i_2R_3 + i_2R_4 = 0$$

Combining the equations for the two meshes, we obtain the following system of equations:

$$\begin{cases} (R_1 + R_2)i_1 - R_2i_2 = v_S \\ R_2 i_1 + (R_2 + R_3 + R_4)\, i_2 = 0 \end{cases}$$

These equations may be solved simultaneously to obtain the desired solution, namely, the mesh currents, i_1 and i_2. One can verify that knowledge of the mesh currents permits determination of all the other voltages and currents in the circuit.

2.9. Matrix equations in electric and electronic engineering

2.10. Nodal and mesh analysis with controlled sources

The methods just described also apply, with relatively minor modifications, in the presence of dependent (controlled) sources. Solution methods that allow for the presence of controlled sources will be particularly useful in the study of transistor amplifiers, transformer circuits, etc. Recall from the Section 1.3 that a dependent source is a source that generates a voltage or current that depends on the value of another voltage or current in the circuit.

When a dependent source is present in a circuit to be analyzed by node or mesh analysis, one can initially treat it as an ideal source and write the node or mesh equations accordingly. In addition to the equation obtained in this fashion, there will also be an equation relating the dependent source to one of the circuit voltages or currents. This **constraint equation** can then be substituted in the set of equations obtained by the techniques of nodal and mesh analysis, and the equations can subsequently be solved for the unknowns.

It is important to remark that once the constraint equation has been substituted in the initial system of equations, the number of unknowns remains unchanged.

Consider, for example, the circuit of Figure 26, which is a simplified model of a bipolar transistor amplifier.

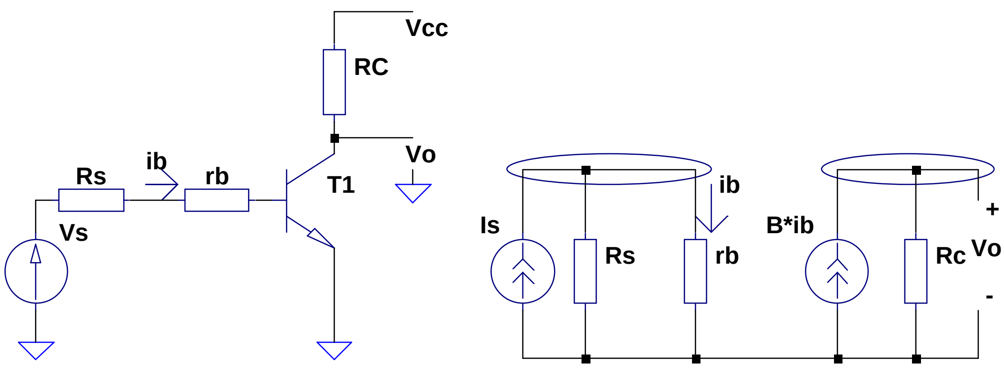


Fig. 26 – Example of the circuit with the current controlled current source

In the circuit of Figure 26, two nodes are easily recognized, and therefore nodal analysis is chosen as the preferred method. Applying KCL at node 1, we obtain the following equation:

$$i_S = v_1\left(\frac{1}{R_S} + \frac{1}{R_b}\right)$$

KCL applied at the second node yields:

$$\beta i_b + \frac{v_2}{R_C} = 0 \ \ (*)$$

Next, it should be observed that the current *ib* can be determined by means of a simple current divider:

$$i_b = i_S \frac{1/R_b}{1/R_b + 1/R_S} = i_S \frac{R_S}{R_b + R_S}$$

which, when inserted in equation (*), yields a system of two equations:

$$i_S = v_1\left(\frac{1}{R_S} + \frac{1}{R_b}\right) - \beta i_S \frac{R_S}{R_b + R_S} = \frac{v_2}{R_C}$$

which can be used to solve for v_1 and v_2.

The techniques presented in this section and the two preceding sections find use more generally than just in the analysis of resistive circuits. These methods should be viewed as general techniques for the analysis of any linear circuit; they provide systematic and effective means of obtaining the minimum number of equations necessary to solve a network problem. Since these methods are based on the fundamental laws of circuit analysis, Kirhhoff voltage law and KCL, they also apply to any electrical circuit, even circuits containing nonlinear circuit elements.

2.11. The principle of superposition

Rather than a precise analysis technique, like the mesh current and node voltage methods, the principle of superposition is a conceptual aid that can be very useful in visualizing the behavior of a circuit containing multiple sources. The principle of superposition applies to any linear system and for a linear circuit may be stated as follows:

In a linear circuit containing *N* sources, each branch voltage and current is the sum of *N* voltages and currents each of which may be computed by setting all but one source equal to zero and solving the circuit containing that single source.

An elementary illustration of the concept may easily be obtained by simply considering a circuit with two sources connected in series, as shown in Figure 27.

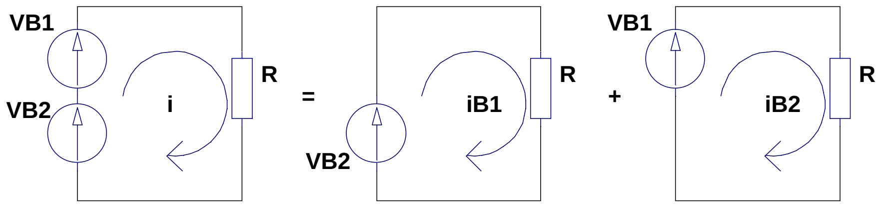


Fig. 27 – The representation of a circuit by two circuit **superposition**

The circuit of Figure 27 is more formally analyzed as follows. The current, i , flowing in the circuit on the left-hand side of Figure 27 may be expressed as:

$$i = \frac{v_{B1} + v_{B2}}{R} = \frac{v_{B1}}{R} + \frac{v_{B2}}{R} = i_{B1} + i_{B2}$$

Figure 27 also depicts the circuit as being equivalent to the combined effects of two circuits, each containing a single source. In each of the two subcircuits, a short circuit has been substituted for the missing battery. This should appear as a sensible procedure, since a short circuit—by definition—will always "see" zero voltage across itself, and therefore this procedure is equivalent to "zeroing" the output of one of the voltage sources.

If, on the other hand, one wished to cancel the effects of a current source, it would stand to reason that an open circuit could be substituted for the current source, since an open circuit is by definition a circuit element through which no current can flow (and which will therefore generate zero current).

The principle of superposition can easily be applied to circuits containing multiple sources and is sometimes an effective solution technique.

3. One-port networks and equivalent circuits

3.1. One-port network

You may recall that, in the discussion of ideal sources in Chapter 1.3, the flow of energy from a source to a load was described in a very general form, by showing the connection of two "black boxes" labeled source and load (see Figure 3,a). Each block—source or load—may be viewed as a two-terminal device, described by an *i-v* characteristic.

The general circuit representation is called a **one-port network** and is particularly useful for introducing the notion of equivalent circuits. Note that the one-port network is completely described by its *i-v* characteristic; this point is best illustrated by the next example.

Problem. Determine the source (load) current i in the circuit of Figure 28 using equivalent resistance ideas.

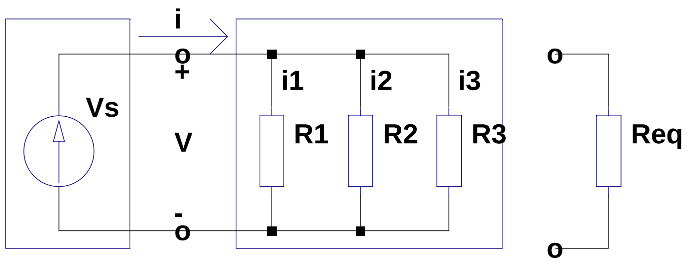


Fig. 28 – Finding the equivalent resistance one-port circuit

Analysis: Insofar as the source is concerned, the three parallel resistors appear identical to a single equivalent resistance of value

$$R_{EQ} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}}$$

Thus, we can replace the three load resistors with the single equivalent resistor R_{EQ}, as shown in Figure 28, and calculate

$$i = \frac{v_S}{R_{EQ}}$$

3.2. Th´evenin and Norton Theorems

In studying node voltage and mesh current analysis, you may have observed that there is a certain correspondence (called **duality**) between current sources and voltage sources, on the one hand, and parallel and series circuits, on the other. This duality appears again very clearly in the analysis of equivalent circuits: it will be shown that equivalent circuits fall into one of two classes, involving either voltage or current sources and (respectively) either series or parallel resistors, reflecting this same principle of duality. The discussion of equivalent circuits begins with the statement of two very important theorems,

The Th´evenin Theorem

As far as a load is concerned, any network composed of ideal voltage and current sources, and of linear resistors, may be represented by an equivalent circuit consisting of an ideal voltage source, v_T , in series with an equivalent resistance, R_T .

The Norton Theorem

As far as a load is concerned, any network composed of ideal voltage and current sources, and of linear resistors, may be represented by an equivalent circuit consisting of an ideal current source, i_N, in parallel with an equivalent resistance, R_N.

Problem

Find the Th´evenin equivalent resistance seen by the load $R6$ in the circuit of Figure 29. ***Given Data:*** $R_1 = 20$; $R_2 = 20$; $I = 5$ A; $R_3 = 10$; $R_4 = 20$; $R_5 = 10$.

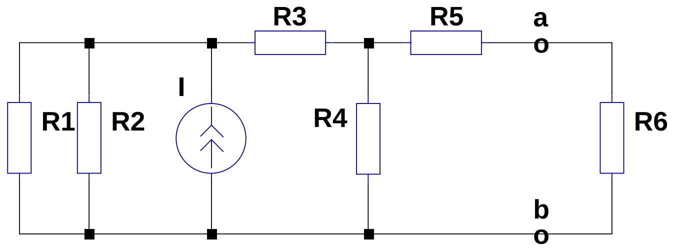


Fig. 29 – A network for which the Th´evenin equivalent resistance is found

Analysis: Following the Th´evenin theorem, we first set the current source equal to zero, by replacing it with an open circuit. Looking into terminal *a-b* we recognize that, starting from the left (away from the load) and moving to the right (toward the load) the equivalent resistance is given by the expression

$R_T = [((R_1||R_2) + R_3) ||R_4] + R_5 = [((20||20) + 10) ||20] + 10 = 20$ Ohm.

Note that the reduction of the circuit started at the farthest point away from the load.

3.3.Computing the Th´evenin voltage

The equivalent (Th´evenin) source voltage v_T is equal to the **open-circuit voltage** present at the load terminals (with the load removed).

This states that in order to compute v_T, it is sufficient to remove the load and to compute the open-circuit voltage at the one-port terminals. Figure 30 illustrates that the open-circuit voltage, v_{OC}, and the Th´evenin voltage, v_T, must be the same if the Th´evenin theorem is to hold.

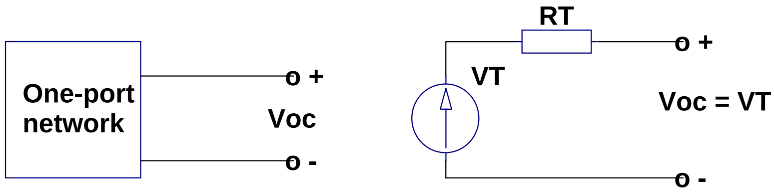


Fig. 30 – Equivalent open circuit

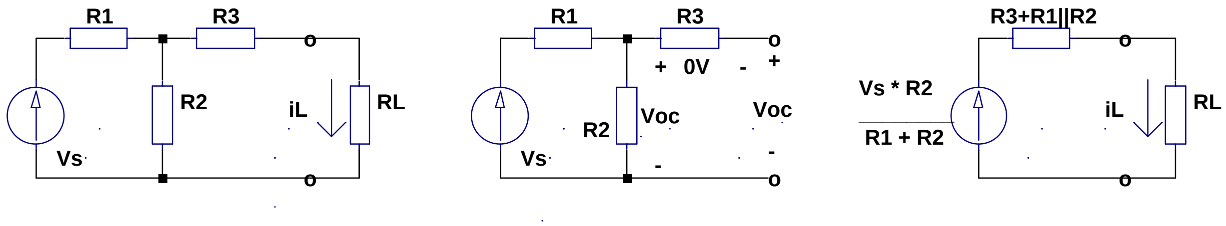
This is true because in the circuit consisting of v_T and R_T, the voltage v_{OC} must equal v_T, since no current flows through R_T and therefore the voltage across R_T is zero. Kirchhoff's voltage law confirms that

$v_T = R_T(0) + v_{OC} = v_{OC}$.

To summarize the main points in the computation of open-circuit voltages, consider the circuit of Figure 31,a. Recall that the equivalent resistance of this circuit was given by

$R_T = R_3 + R_1||R_2$.

To compute v_{OC}, we disconnect the load, as shown in Figure 31,b, and immediately observe that no current flows through R_3, since there is no closed circuit connection at that branch. Therefore, v_{OC} must be equal to the voltage across R_2, as illustrated in Figure 31,b.



a) b) c)

Fig. 31 – Equivalent network finding

Since the only closed circuit is the mesh consisting of v_S, R_1, and R_2, the answer we are seeking may be obtained by means of a simple voltage divider:

$$v_{OC} = v_{R2} = v_S \frac{R_2}{R_1 + R_2}$$

It is instructive to review the basic concepts outlined in the example by considering the original circuit and its Th´evenin equivalent side by side, as shown in Figure 31. The two circuits of Figure 30 are equivalent in the sense that the current drawn by the load, i_L, is the same in both circuits, that current being given by:

$$i_L = v_S \frac{R_2}{R_1 + R_2} \cdot \frac{1}{(R_3 + R_1 \| R_2) + R_L} = \frac{v_T}{R_T + R_L}$$

3.4.Computing the Norton Current

The computation of the Norton equivalent current is very similar in concept to that of the Th´evenin voltage. The following definition will serve as a starting point:

The Norton equivalent current is equal to the **short-circuit current** that would flow were the load replaced by a short circuit. An explanation for the definition of the Norton current is easily found by considering, again, an arbitrary one-port network, as shown in Figure 32, where the one-port network is shown together with its Norton equivalent circuit.

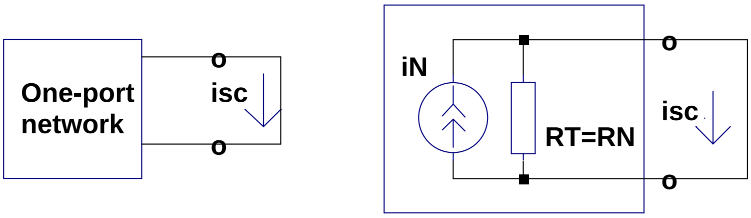


Fig. 32. – To derive the Norton equivalent current

It should be clear that the current, i_{SC}, flowing through the short circuit replacing the load is exactly the Norton current, i_N, since all of the source current in the circuit of Figure 32 must flow through the short circuit.

Consider the circuit on Figure 33, shown with a short circuit in place of the load resistance. Any of the techniques presented in this chapter could be employed to determine the current i_{SC}. In this particular case, mesh analysis is a convenient tool, once it is recognized that the short-circuit current is a mesh current. Let i_1 and $i_2 = i_{SC}$ be the mesh currents in the circuit of Figure 33. Then, the following mesh equations can be derived and solved for the short-circuit current:

$$\begin{cases} (R_1 + R_2)i_1 - R_2 i_{SC} = v_S; \\ -R_2 i_1 + (R_2 + R_3) i_{SC} = 0. \end{cases}$$

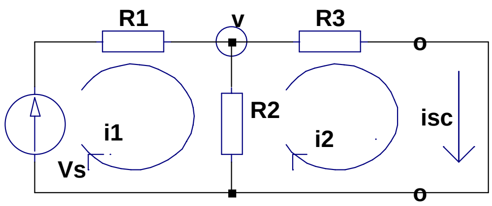


Fig. 33 – Example of the Norton equivalent current deriving

An alternative formulation would employ nodal analysis to derive the equation

$$\frac{v_S - v}{R_1} = \frac{v}{R_2} + \frac{v}{R_3}$$

leading to

$$v = v_S \frac{R_2 R_3}{R_1 R_3 + R_2 R_3 + R_1 R_2}$$

Recognizing that $i_{SC} = v/R_3$, we can determine the Norton current to be:

$$i_N = \frac{v}{R_3} = \frac{v_S R_2}{R_1 R_3 + R_2 R_3 + R_1 R_2}$$

Thus, conceptually, the computation of the Norton current simply requires identifying the appropriate short-circuit current.

3.5. Duality of the Norton and Th´evenin equivalent circuits

The Norton and Th´evenin theorems state that any one-port network can be represented by a voltage source in series with a resistance, or by a current source in parallel with a resistance, and that either of these representations is equivalent to the original circuit, as illustrated in Figure 34.

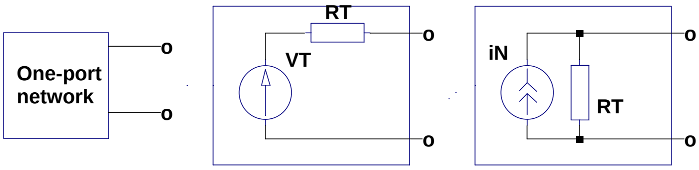


Fig. 34. One-port network, and its equivalent circuits

An extension of this result is that any circuit in Th´evenin equivalent form may be replaced by a circuit in Norton equivalent form, provided that we use the following relationship:

$$v_T = R_T\, i_N\,.$$

3.6. Experimental Determination of Th´evenin and Norton Equivalents

The idea of equivalent circuits as a means of representing complex and sometimes unknown networks is useful not only analytically, but in practical engineering applications as well. It is very useful to have a measure, for example, of the equivalent internal resistance of an instrument, so as to have an idea of its power requirements and limitations. Fortunately, Th´evenin and Norton equivalent circuits can also be evaluated experimentally by means of very simple techniques. The basic idea is that the Th´evenin voltage is an open-circuit voltage and the Norton current is a short-circuit current. It should therefore be possible to conduct appropriate measurements to determine these quantities. Once v_T and i_N are known, we can determine the Th´evenin resistance of the circuit being analyzed according to the relationship

$$R_T = v_T / i_N$$

Figure 35 illustrates the measurement of the open-circuit voltage v_T and shortcircuit current i_N for an arbitrary network connected to any load and also illustrates that the procedure requires some special attention, because of the nonideal nature of any practical measuring instrument. The figure clearly illustrates that in the presence of finite meter resistance, r_m, one must take this quantity into account in the computation of the short-circuit current and open-circuit voltage; v_{OC}

and i_{SC} appear between quotation marks in the figure specifically to illustrate that the measured "open-circuit voltage" and "short-circuit current" are in fact affected by the internal resistance of the measuring instrument and are not the true quantities.

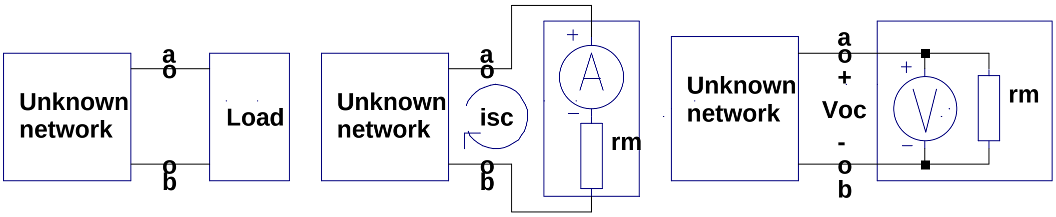


Fig. 35. Experimental estimation of the unknown network parameters

One can verify that the following expressions for the true short-circuit current and open-circuit voltage apply

$$i_N = "i_{SC}"\left(1 + \frac{r_m}{R_T}\right)$$

$$v_T = "v_{OC}"\left(1 + \frac{R_T}{r_m}\right)$$

where i_N is the ideal Norton current, v_T the Th´evenin voltage, and R_T the true Th´evenin resistance. For an ideal ammeter, r_m should approach zero, while in an ideal voltmeter, the internal resistance should approach an open circuit (infinity); thus, the two expressions just given permit the determination of the true Th´evenin and Norton equivalent sources from an (imperfect) measurement of the open-circuit voltage and short-circuit current, provided that the internal meter resistance, r_m, is known. Note also that, in practice, the internal resistance of voltmeters is sufficiently high to be considered infinite relative to the equivalent resistance of most practical circuits; on the other hand, it is impossible to construct an ammeter that has zero internal resistance. If the internal ammeter resistance is known, however, a reasonably accurate measurement of short-circuit current may be obtained.

The following example illustrates the point.

Problem: Determine the Th´evenin equivalent of an unknown circuit from measurements of open-circuit voltage and short-circuit current. ***Given Data—*** Measured v_{OC} = 6.5 V; Measured i_{SC} = 3.75 mA; r_m = 15 .

Analysis— The unknown circuit, shown on the top left in Figure 36, is replaced by its Th´evenin equivalent, and is connected to an ammeter for a measurement of the short-circuit current, and then to a voltmeter for the measurement of the open-circuit voltage . The open-circuit voltage measurement yields the Th´evenin voltage: $v_{OC} = v_T$ = 6.5 V.

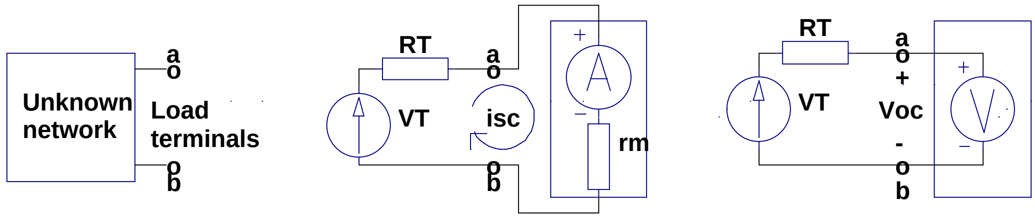


Fig. 36. Practical measurement of the unknown circuit parameters

To determine the equivalent resistance, we observe in the figure depicting the voltage measurement that, according to the circuit diagram,

$$\frac{v_{OC}}{i_{SC}} = R_T + r_m$$

Thus,

$$R_T = \frac{v_{OC}}{i_{SC}} - r_m = 1{,}733 - 15 = 1{,}718\Omega$$

Note, that in most cases, it is not advisable to actually shortcircuit a network by inserting a series ammeter as shown in Figure 36; permanent damage to the circuit or to the ammeter may be a consequence.