Computer reliability

Laboratory exersize 1

Reliability parameters of computers

1.Goal: To achieve knowledges about the general reliability parameters like success distribution, fault distribution, failure rate, density function, etc.

2.Theoretical basics.

The main reliability characteristic is the **success distribution function**,or shortly **reliability**, which is equal to the probability of the reliable computer operation in the given time interval t by the given operation conditions

$$P(t)=W\{T>t\},$$

where T is time amount of the reliable system operation, W{A} is the probability of the event A. The opposite meaning function is **failure distribution function,** or shortly **unreliability**

$$Q(t)=W\{T\le t\}.$$

From these function equations the next relations are followed directly

$$P(t)+Q(t)=1,$$

$$0\le P(t)\le 1,\ \ 0\le Q(t)\le 1,\qquad P(t_2)\le P(t_1),\ \ Q(t_2)\ge Q(t_1),\ \text{where}\ t_2>t_2.$$

In the reliability theory the systems are considered which satisfy the equations

$$P(0)=1,\ \ Q(0)=0,\ \ P(\infty)=0,\ \ Q(\infty)=1,$$

i.e. these systems are operable at the initial time, and the operation time is finite one.

The determinant of the failure distribution function is the distribution density or shortly **density function**

$$f(t)=\frac{dQ(t)}{dt}=-\frac{dP(t)}{dt}.$$

The relative value of the density function is the **fault intensity function** or **failure rate**

$$\lambda(t)=\frac{f(t)}{P(t)}.$$

When taking the integral of the density function we derive the estimation of the success distribution as the following

$$P(t)=e^{-\int_0^t \lambda(t)dt}$$

If a system consists of 3 parts with the failure rates λ_1, λ_2, λ_3, and a system fails when any of the parts fails then the resulting failure rate of the system is

$$\lambda=\lambda_1+\lambda_2+\lambda_3.$$

Operation period to a fault is equal to the **mean time to failure**:

$$T_0 = \int_0^\infty t \cdot f(t)dt = \int_0^\infty P(t)dt.$$

if λ=const then

$$P(t) = e^{-\lambda t},$$

and as a result

$$T_0 = \int_0^\infty e^{-\lambda t}dt = \frac{1}{\lambda}.$$

The system reliability $P(t/t_1)$ at the interval (t_1, t), if the system till t_1 was in the faultless operation is searched from the relation

$$P(t) = P(t_1) \cdot P(t/t_1)$$

Hence the **reliability of faultless operation at the interval** (t_1, t) is

$$P(t/t_1) = e^{-\int_{t_1}^{t} \lambda(t)dt}.$$

If λ=const , which is named as the **exponential fault distribution law**, or **constant hazard rate law** then $P(t/t_1) = e^{-\lambda(t-t_1)}$. In this situation the probability of the faultless operation is independed on the previous operation. It is true for a system without runout and consenescence, or when the time period is small.

The probability of the faultless operation for the time period $t_0 << T_0$ by the exponential fault distribution law is equal to

$$P(t_0) = e^{-\lambda \cdot t_0} \approx 1 - \lambda t_0.$$

Respectively the probability of a fault for a time period, or the **effective failure rate** is

$$\lambda_e = \frac{1 - P(t_0)}{t_0}.$$

The **guaranteed technical resource** t_γ with respect to the guaranteed probability γ is derived from the equation $P(t_\gamma) = \gamma$. By λ=const the equation is $e^{-\lambda t_\gamma} = \gamma$ from which we derive $t_\gamma = -T_o \cdot \ln g$. Because the value γ is near 1 (for example, γ=0,9), then

$$t_\gamma = -T_0 \cdot \ln(1 - (1 - \gamma)) \approx (1 - \gamma) \cdot T_0.$$

In the real world the exponential fault distribution law, or shortly E-law occurs rarely. Therefore, to estimate the reliability parameters of the real systems more complex laws are considered. The **diffused monotonous distribution** (**hazard**) **model**, or shortly **DM-model** is used for the systems which fault due to the wearing, corrosion, etc., and is described by the formulas

$$f(t) = \frac{t + \tau}{2\nu t\sqrt{2\pi\tau t}} e^{-\frac{(t-\tau)^2}{2\nu^2\tau t}},$$

$$P(t) = 1 - \int_0^t f(x)dx = 1 - \int_0^t \frac{x + \tau}{2\nu x\sqrt{2\pi\tau x}} e^{-\frac{(x-\tau)^2}{2\nu^2\tau x}} dx, \qquad T_o = \tau(1 + \frac{\nu^2}{2}),$$

where τ is the **scale parameter** and $\nu > 0$ is the shape parameter. Usually the variable substitution $\frac{x-\tau}{\nu\sqrt{\tau x}} = u$ is used. Then $du = \frac{x+\tau}{2\nu x\sqrt{\tau x}}dx$, and the integration limits from 0 to t are substituted by limits from $-\infty$ to $\frac{t-\tau}{\nu\sqrt{\tau x}}$. As a result, the formula is simplified as

$$P(t) = \Phi\left(\frac{t-\tau}{\nu\sqrt{\tau t}}\right), \text{ where } \Phi(x) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{x} e^{-\frac{u^2}{2}}du \ .$$

DN is the **diffused nonmonotonous distribution model** which is used for the electronic systems, which fails from the oldness, electromigration, etc.:

$$f(t) = \frac{\sqrt{\tau}}{\nu t\sqrt{2\pi t}}e^{-\frac{(t-\tau)^2}{2\nu^2\tau t}}, \ P(t) = \Phi\left(\frac{\tau-t}{\nu\sqrt{\tau t}}\right) - e^{2\nu^{-2}}\Phi\left(-\frac{\tau+t}{\nu\sqrt{\tau t}}\right), \text{ and } T_O = \tau.$$

LN is the **logarithmically normal distribution model** which is used for the electronic and electromechanic systems, which fail due to the weariness from the periodical loading stresses:

$$f(t) = \frac{1}{\nu t\sqrt{2\pi}}e^{-\frac{(\ln t-\ln\tau)^2}{2\nu^2}}, \ P(t) = \Phi\left(\frac{\ln\tau-\ln t}{\nu}\right), \text{ and } T_O = \tau e^{\frac{\nu^2}{2}}.$$

W is the **Weibull hazard model**, which is used as the approximation of different density and reliability functions depending on the scale parameter τ and shape parameter ν :

$$f(t) = \frac{\nu}{\tau}\left(\frac{t}{\tau}\right)^{\nu-1}e^{-\left(\frac{t}{\tau}\right)^{\nu}}, \ P(t) = e^{-\left(\frac{t}{\tau}\right)^{\nu}}, \ \lambda(t) = \frac{\nu}{\tau}\left(\frac{t}{\tau}\right)^{\nu-1}, \ T_O = \tau\Gamma\left(1+\frac{1}{\nu}\right),$$

where $\Gamma(x) = \int_0^{\infty} u^{x-1}e^{-u}du$ is the gamma function for $x>0$.

3.Tasks

Task 1.

Consider a system consists of n processing units (PU) and it fails when n-k and more PUs fail. The failures are independent and have equal and independent failure rate $\lambda = 10^{-4}$ hours^{-1}.

Build the following 2-d diagrams of the argument λt both for a single PU and for the whole system:

A1) success distribution function;

A2) failure distribution function;

A3) density function;

A4) failure rate;

A5) mean time to failure;

A6) reliability of faultless operation at the interval (τ, t+ τ) if till the moment $\tau = 10^4$ hours the system has operated without faults.

A7) reliability of faultless operation at the interval (τ, t+ τ) if till the moment τ m PUs had failed (a single PU is not considered).

Estimate the following numerical characteristics both of a single PU and of the whole system

B1) mean time to failure;

B2) effective failure rate;

B3) mean time to failure if a system (orPU) has operated $\tau=10^4$ hours without faults.

B4) mean time to failure if a system (orPU) has operated $\tau=10^4$ hours, and r PUs have failed.

B5) and B6) guaranteed technical resource t_γ with respect to the guaranteed probability γ=0.9 and γ=0.99.

Estimate a number of additional PUs which are needed for

C1) decreasing λ_e in M times;

C2) increasing mean time to failure in 2 times.

Task 2.

Perform all the points of the task 1 for the situation when the distribution function is not exponential but Q(t): DM,DN,LN, or W.

4.Variant selection

The individual parameters for the tasks are selected in the following table

C_5	n	Q(t)	τ, hours	ν	C_4	M	C_3	K	C_2	m	r
0	9	DM	6667	1	0	10	0	0	0	1	2
1	8	DN	10^4	1	1	100	1	1	1	2	1
2	7	LN	6075	1	2	1000	2	2	-	-	-
3	6	W	5000	0,5	3	10000	-	-	-	-	-
4	5	W	11077	1,5	-	-	-	-	-	-	-

where C_k is the remainder of division of the student's record book number to k.

For example, consider the record book number k=1234.

Then k = 246*5+4 = 308*4+2 = 411*3+1 = 617*2+0.

As a result, C_5 = 4; C_4 = 2; C_3 = 1; C_2 = 0.

5.Example of the task solution

Consider a system of 8 PUs which failes when 6,7 or 8 PUs failed. The faults are independent on each other, and have equal and stable failure rate. Estimate the system reliability if a system has operated τ hours without faults.

The reliability is derived from the formula

$$P_C(t+\tau)=P_C(\tau)\cdot P_C(t+\tau/\tau),$$

where $P_C(t)$ is reliability of the system, $P_C(t+\tau/\tau)$ is reliability to find.

Therefore, $$P_c(t+\tau/\tau)=\frac{P_c(t+\tau)}{P_c(\tau)}.$$

Because the system fails when 6, 7, or 8 PUs fail, then

$$P_c(t)=1-Q_c(t)=1-C_8^6P^2(1-P)^6-C_8^7P(1-P)^7-(1-P)^8=$$
$$=1-(1-P)^6(1+6P+21P^2),$$

where $P=e^{-\lambda t}$ is reliability of a single PU, C_m^n is a combination of n from m. The result is

$$P_c(t+\tau/\tau)=\frac{1-(1-P(t+\tau))^6(1+6P(t+\tau)+21P^2(t+\tau))}{1-(1-P(\tau))^6(1+6P(\tau)+21P^2(\tau))}.$$

Consider the same system with the same conditions. Estimate the system reliability at the period from τ to τ+t hours if in the system which was operated τ hours 2 PUs failed.

Because the failure rate is considered to be stable, then the system reliability doesn't depend on the time period till the moment τ , but it depends on the interval (τ, τ+t). Therefore the system will fail, when 6 and more PUs fail in it. But 2 PUs have already failed, and as a result, the system will fail if 4, 5, or 6 PUs fail. The resulting reliability is equal to

$$P_C(t) = 1 - C_6^4 P^2(1-P)^4 - C_6^5 P(1-P)^5 - (1-P)^6 = 1\text{-}15P^2(1\text{-}P)^4\text{-}6P(1\text{-}P)^5\text{-}(1\text{-}P)^6.$$